Cuyamaca College
Math 280

Name _____
Instructor: Dan Curtis

Practice Final Exam

1. Evaluate the integral.

$$\int \sin^3 x \, dx$$

2. Evaluate the integral.

$$\int \frac{5x+1}{x^2+3x+2} \, dx$$

3. Evaluate the integral.

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

4. Evaluate the integral.

$$\int e^{2x} \cos(x)\, dx$$

5. Evaluate the integral or show that it diverges.

$$\int_0^\infty xe^{-x}\, dx$$

6. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{3n}{e^n}$$

7. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - 5n - 3}$$

8. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\pi^n}{3^n}$$

9. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{2 \ln n}{n}$$

10. Determine whether the series is conditionally convergent, absolutely convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n + 5}$$

11. Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{6^{n+1}}{10^n}$$

12. Find the interval and radius of convergence for the series.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n2^n}$$

13. Use the Maclaurin series for $f(x) = e^x$ to find the sum of the series.

$$\sum_{n=1}^{\infty} \frac{3(-1)^n}{n!}$$

14. Find the power series for the function f.

$$f(x) = \frac{1}{2 + x^2}$$

15. Find the second degree Taylor Polynomial, T_2, at $a = 0$.

$$f(x) = xe^{x/2}$$

16. Use the Maclaurin series to evaluate the integral.

$$\int \cos(x^2)\, dx$$

17. Find the Taylor series for $f(x) = \ln x$ centered at $a = 1$.

18. a) Eliminate the parameter to find a Caresian equation of the curve.

    b) Sketch the curve and indicate its orientation.

$$x = t - 3, \quad y = \sqrt{t}, \quad t \geq 0$$

19. Find an equation of the tangent line to the curve at the point corresponding to $t = 1$.

$$x = 5t - t^3, \quad y = t^2 - 5t$$

20. Find the area enclosed by one loop of the curve.

$$r = 4\sin(3\theta)$$

21. Find a Cartesian equation for the polar curve and use it to sketch the graph of the curve.

$$r = 6\cos\theta + 8\sin\theta$$

22. Find the exact length of the curve.

$$r = \theta^2, \quad 0 \leq \theta \leq 1$$

Solutions:

1. $-\cos x + \frac{1}{3}\cos^3 x + C$
2. $-4\ln|x+1| + 9\ln|x+2| + C$
3. $-\dfrac{\sqrt{4-x^2}}{4x} + C$
4. $\dfrac{2e^{2x}\cos x}{5} + \frac{e^{2x}\sin x}{5} + C$
5. 1
6. Convergent by the root or ratio test
7. Convergent by the limit comparison test
8. Divergent by test for divergence or geometric series
9. Divergent by integral test or comparison test
10. Absolutely convergent by the comparison test
11. 15
12. $[1, 5)$, $R = 2$
13. $\frac{1}{e}$
14. $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}x^{2n-2}}{2^n}$
15. $T_2 = x + \dfrac{x^2}{2}$
16. $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$
17. $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n}(x-1)^n$
18. a) $x = y^2 - 3, \quad y \geq 0$
19. $y = -\frac{1}{2}x + 2$
20. $\frac{4\pi}{3}$
21. $x^2 + y^2 = 6x + 8y$

    By completing the square, we get $(x-3)^2 + (y-4)^2 = 25$, which is a circle centered at $(3, 4)$ with radius $r = 5$.
22. $\frac{1}{2}(5\sqrt{5} - 8)$